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TECHNICAL NOTES

Interaction of surface suction/blowing with buoyancy force on mixed convection flow adjacent to an inclined flat plate

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INTRODUCTION

The buoyancy effect on an inclined surface possesses two components with one acting along the flow direction and the other normal to the plate. For convenience, the former is called Buoyancy A and the latter Buoyancy B. The characteristics of Buoyancy A and Buoyancy B are quite different in nature. For example, in conventional non-similarity transformation, the non-similarity variable is defined by $\xi = Gr_x/Re_x^2$ for a vertical flat plate [1] when Buoyancy A dominates the buoyancy alone, but defined by $\xi = Gr_x/Re_x^{5/2}$ for a horizontal flat plate [2] when Buoyancy B is the only effective buoyancy component. On an inclined flat plate, however, both Buoyancy A and Buoyancy B must be taken into account. Under such a situation, the non-similarity variable ξ becomes very difficult to define.

The work of Tsuruno and Iguchi [3] seems to be the only investigation so far that studies the effect of surface suction/blowing on mixed convection along a vertical flat plate. In their study, Tsuruno and Iguchi [3] employed the conventional definition $\xi = Gr_x/Re_x^2$ for the non-similarity variable, while a new parameter $C = v_w[u_\infty/v\beta g(T_w - T_\infty)]^{1/2}$ was proposed for investigating the effect of surface suction/blowing. Unfortunately, their analysis does not apply to the case of pure forced convection because the characteristic buoyancy force $\beta g(T_w - T_\infty)$ appears in the denominator of parameter C.

In the present investigation, mixed convection in the presence of surface suction/blowing on an inclined flat plate is studied. A new transformation is proposed such that there is no singularity in the transformed governing equations and the associated boundary conditions. Thus, unlike in Tsuruno and Iguchi [3], the present analysis reduces to that for pure forced convection with surface suction/blowing when the buoyancy parameter is assigned to zero. In addition, it applies to mixed convection on an inclined flat plate as well when the surface suction/blowing parameter becomes zero.

THEORETICAL ANALYSIS

Consider a laminar forced flow of uniform velocity u_∞ and temperature T_∞ aligned parallel to an inclined flat plate having an acute angle γ measured from the horizontal. The surface of the plate is maintained at a constant temperature T_w and has a uniform blowing velocity v_w normal to the surface. As demonstrated in ref. [4], by introducing the general transformation

$$\psi = vxf/\delta, \quad \eta = y/\delta, \quad \theta = (T - T_\infty)/(T_w - T_\infty)$$
$$\xi = \xi(x), \quad \delta = \delta(x), \quad f = f(\xi, \eta) \quad (1)$$

the conservation equations for the problem under consideration become

$$f''' + aff'' + bf'^2 + A\theta + B[p\eta\theta + pw - c\partial w/\partial\xi] = c(f''\partial f/\partial\xi - f'\partial f'/\partial\xi) \quad (2)$$

$$\theta'' + Pr\,af\theta' = Pr\,c(\theta'\partial f/\partial\xi - f'\partial\theta/\partial\xi) \quad (3)$$

$$w' + \theta = 0 \quad (4)$$

where

$$a = 1 - p, \quad b = 2p - 1, \quad c = -m\xi$$
$$p = (x/\delta)(\mathrm{d}\delta/\mathrm{d}x), \quad m = (x/\xi)(\mathrm{d}\xi/\mathrm{d}x)$$
$$A = \sigma|Gr_x|(\delta/x)^4 \sin\gamma, \quad B = \sigma|Gr_x|(\delta/x)^5 \cos\gamma$$
$$Gr_x = \beta g(T_w - T_\infty)x^3/v^2. \quad (5)$$

The associated boundary conditions are

$$(p-1)f(\xi, 0) + c\partial f(\xi, 0)/\partial\xi = v_w\delta/v \quad (6)$$
$$f'(\xi, 0) = 0, \quad f'(\xi, \infty) = u_\infty\delta^2/vx \quad (7)$$
$$\theta(\xi, 0) - 1 = \theta(\xi, \infty) = w(\xi, \infty) = 0. \quad (8)$$

In equations (2)–(8), the primes denote partial differentiation with respect to η. The index $\sigma = 1$ stands for buoyancy assisting flow and $\sigma = -1$ for buoyancy opposing flow. The function $\delta(x)$ to be defined is the characteristic boundary layer thickness. Upon studying equations (5) and (7), one sees that letting $\delta = (vx/u_\infty)^{1/2}$ would lead to $f'(\xi, \infty) = 1$ from equation (7) and $a = 1/2$ and $b = 0$ from equations (5). Based on such a definition for $\delta(x)$, the coefficients of Buoyancy A and Buoyancy B reduce, respectively, to

$$A = \sigma(|Gr_x|/Re_x^2)\sin\gamma$$
$$B = \sigma(|Gr_x|/Re_x^{5/2})\cos\gamma \quad (9)$$

and boundary condition (6) becomes

$$2m\xi\partial f(\xi, 0)/\partial\xi + f(\xi, 0) = -2(v_w/u_\infty)Re_x^{1/2}. \quad (10)$$

Three nonsimilarities could arise from equations (9) and (10). Unfortunately, there is only one arbitrary variable $\xi(x)$. This means that two additional parameters are needed to properly handle the three nonsimilarities. For mixed convection along an inclined flat plate with uniform surface suction/blowing considered here, it might be best to define a buoyancy parameter $\Omega = |Gr_x|/Re_x^3 = \beta g|T_w - T_\infty|v/u_\infty^3$ for the buoyancy effect and a surface suction/blowing parameter $I = v_w/u_\infty$ for the effect of surface suction/blowing. With the aid of Ω and I, the non-similarity variable $\xi(x)$ can be defined by $\xi = Re_x^{1/8}$ for the only nonsimilarity Re_x that remains in the system of equations. The power 1/8 employed here is for an adaptive grid system in the flow direction, i.e. $0 \leqslant \xi \leqslant 5$ corresponds to $0 \leqslant Re_x \leqslant 3.9 \times 10^5$. With this, boundary condition (10) reduces to $f(\xi, 0) = -I\xi^4$ and the parameters have the values $m = 1/8$, $A = \sigma\Omega\xi^8 \sin\gamma$ and $B = \sigma\Omega\xi^4 \cos\gamma$.

It is noteworthy that there is no singularity in the present transformation. Thus, equations (2)–(4) will reduce to that

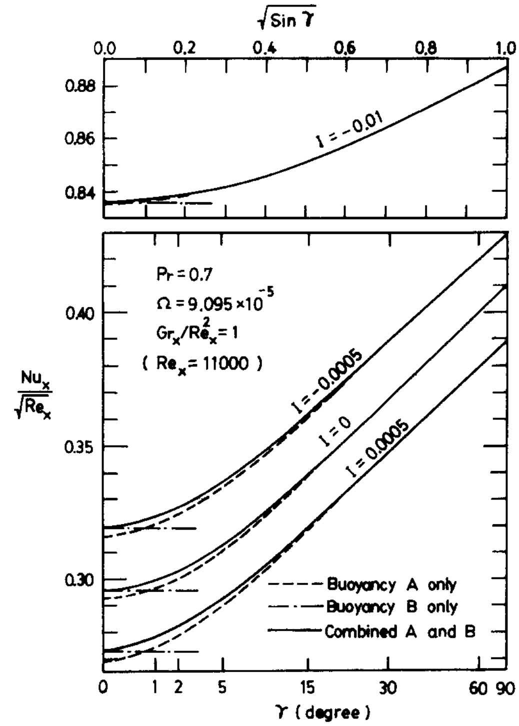

FIG. 1. Characteristics of Buoyancy A and Buoyancy B on an inclined flat plate in the presence of surface suction/blowing for $Pr = 0.7$.

for pure forced convection with surface suction/blowing when the buoyancy parameter Ω is assigned zero. On the other hand, they apply to mixed convection along an inclined flat plate without surface suction/blowing as parameter I becomes zero. Once equations (2)–(4) are solved, the Nusselt number expression $Nu_x/Re_x^{1/2}$ and the friction factor $(C_f)_x$ can be determined, respectively, from

$$Nu_x/Re_x^{1/2} = -\theta'(\xi, 0) \quad (11)$$

$$(C_f)_x Re_x^{1/2} = 2f''(\xi, 0) \quad (12)$$

as done by previous investigators.

RESULTS AND DISCUSSION

The system of governing equations (2)–(8) was solved by using the weighting function method proposed in ref. [5] for Prandtl numbers of 0.7 and 7. Computations were performed for the entire range of inclination $0° \leqslant \gamma \leqslant 90°$ with a surface suction/blowing parameter I ranging from -0.01 to 0.01 that covers a wide range for both cases of suction ($I < 0$) and blowing ($I > 0$). A representative value of the buoyancy parameter of $\Omega = 9.095 \times 10^{-5}$ is employed for both buoyancy assisting ($\sigma = 1$) and opposing ($\sigma = -1$) flows. The step size $\Delta\xi = \Delta\eta = 0.05$ along with $\eta_\infty = 10$ was found to be adequate for all parameters that were investigated in the present study. All computations were carried out on a CDC Cyber 840 computer.

Figure 1 reveals the characteristics of Buoyancy A and B with and without the surface suction/blowing. The results of the Nusselt number expression $Nu_x/Re_x^{1/2}$ are presented as a function of $\sin^{1/2}\gamma$ (see the upper abscissa) for the case of $Pr = 0.7$, $\Omega = 9.095 \times 10^{-5}$ and $Gr_x/Re_x^2 = 1$. The solid curves give the results that consider the combined effect of Buoyancy A and B whereas the dashed curves give the results that consider only Buoyancy A. Therefore, the difference between the solid and the dashed curves arises from the contribution of Buoyancy B. From Fig. 1, it is seen that for $Re_x = 11\,000$ (i.e. $Re_x = (Gr_x/Re_x^2)/\Omega$), Buoyancy B has a maximum effect at $\gamma = 0°$ and essentially becomes negligible at $\gamma > 15°$. In the inclination of $15° \leqslant \gamma \leqslant 90°$, the Nusselt number seems to possess a linear variation with respect to $\sin^{1/2}\gamma$. The effect of Buoyancy B vanishes when the surface suction is strong, say $I = -0.01$. Note that the effect of Buoyancy A disappears at $\gamma = 0°$. Hence, the dashed curves at $\gamma = 0°$ denote the result for pure forced convection. This implies that the maximum effect of Buoyancy A can be evaluated from the difference between $\gamma = 90°$ and $0°$ on the dashed curve. For example, the maximum effect of Buoyancy A is $0.4087 - 0.2927 = 0.1160$ for $I = 0$. This effect reduces to $0.8860 - 0.8359 = 0.0501$ as a surface suction of $I = -0.01$ is applied on the flat plate. Therefore, a surface suction decreases both effects of Buoyancy A and B. Nevertheless, surface suction increases the overall heat transfer rate in spite of its repression on the buoyancy effect.

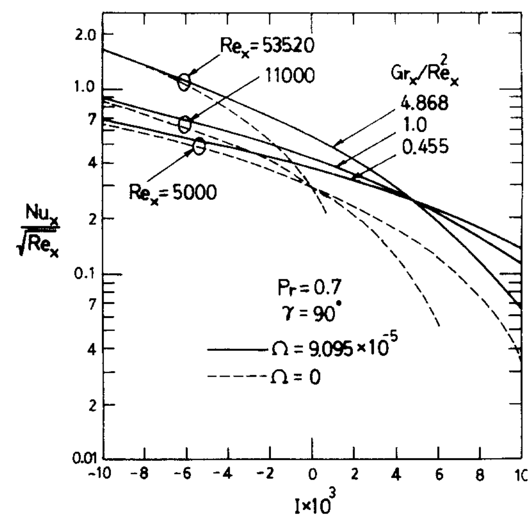

FIG. 2. Interaction of surface suction/blowing with buoyancy force on a vertical flat plate for $Pr = 0.7$.

The interaction of surface suction/blowing with buoyancy force on a vertical flat plate ($\gamma = 90°$) is presented in Fig. 2 for $Pr = 0.7$ and $Gr_x/Re_x^2 = 0.4547$, 1 and 4.868 while the buoyancy parameter is maintained at $\Omega = 9.095 \times 10^{-5}$. The parameters $Gr_x/Re_x^2 = 0.4547$, 1 and 4.868 thus correspond, respectively, to $Re_x = 5000$, 11 000 and 53 520. For comparison, the results for pure forced convection ($\Omega = 0$) obtained by Sparrow and Yu [6] are also plotted in Fig. 2 with dashed curves. For pure forced convection, the heat transfer is enhanced by a surface suction ($I < 0$). Such an enhancement in heat transfer is amplified as the value of Re_x increases. The trend is reversed for the surface blowing case ($I > 0$). Thus, there exists a common crossing among the dashed curves as can be observed from Fig. 2. In fact, this crossing point stands for the Blasius flow because both I amd Ω are zero there. It is interesting to note from Fig. 2 that this particular point moves from $(I, Nu_x/Re_x^{1/2}) = (0, 0.2927)$ to a location near (0.004895, 0.2425) as the buoyancy parameter Ω increases from 0 to 9.095×10^{-5}. This means that the value of the Nusselt number expression $Nu_x/Re_x^{1/2}$ decreases downstream (i.e. Re_x increases) if the strength of the surface blowing is beyond the critical value, i.e. $I > 0.004895$. In contrast, the value of $Nu_x/Re_x^{1/2}$ increases if $I < 0.004895$.

This critical point has been predicted also by Tsuruno and Iguchi [3] in a coordinate system of $2Nu_x/Re_x^{1/2}$ vs C with $C = v_w[u_\infty/v\beta g(T_w - T_\infty)]^{1/2} = I\Omega^{-1/2}$ being the surface suction/blowing parameter. The critical point was found to exist at $(C, 2Nu_x/Re_x^{1/2}) = (0.467, 0.508)$ for the case of $Pr = 0.72$. Based on their finding, Tsuruno and Iguchi [3] concluded that the buoyancy force enhances the heat transfer for $C < 0.467$, and vice versa for $C > 0.467$. However, one should

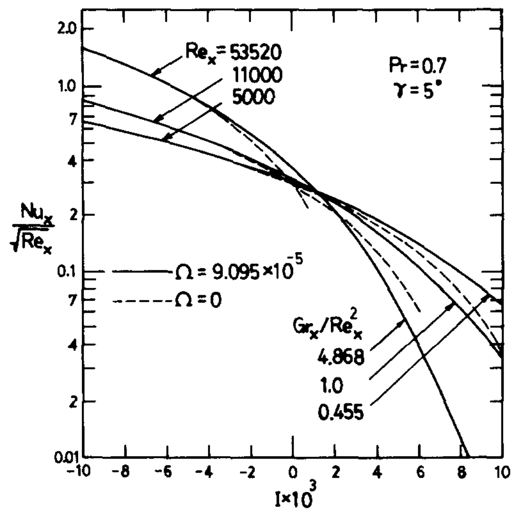

FIG. 3. Interaction of surface suction/blowing with buoyancy force on an inclined flat plate of small inclination ($\gamma = 5°$) for $Pr = 0.7$.

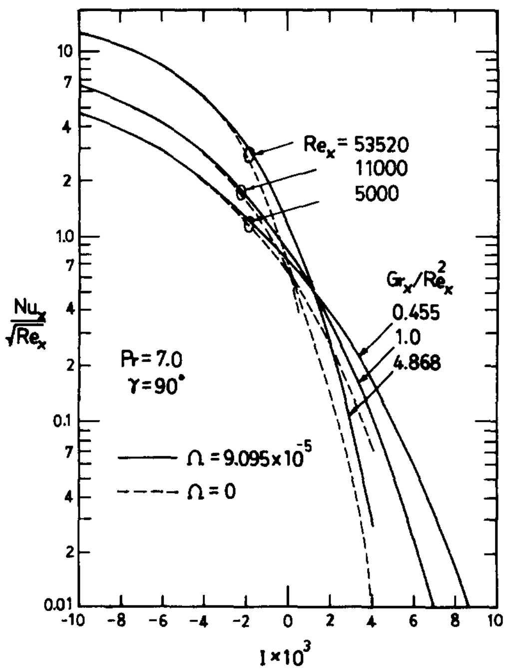

FIG. 4. Interaction of surface suction/blowing with buoyancy force on a vertical flat plate for $Pr = 7$.

note that it is impossible to increase the buoyancy force without changing the value of C. Therefore, their conclusion is not relevant. In the present investigation, the surface suction/blowing parameter I is defined by v_w/u_∞ such that the effect of buoyancy force can be properly observed. In a comparison between the results for $\Omega = 0$ and for 9.095×10^{-5} shown in Fig. 2, one sees that the buoyancy force always enhances the heat transfer (at a given location Re_x) for the entire surface condition regime from suction to blowing. Similar phenomena as that in Fig. 2 can also be found in Fig. 3 for $\gamma = 5°$, a flat plate with small inclination.

Figure 4 reveals the interaction of surface suction/blowing with buoyancy force on a vertical flat plate for $Pr = 7$ and $\Omega = 9.095 \times 10^{-5}$. As can be seen from Fig. 4, surface suction/blowing has a quite sensitive effect on the heat transfer performance for $Pr = 7$. Thus, a critical point is found at a small value of I (i.e. $I = 0.001074$). As a final note, it is mentioned that Figs. 2 and 4 are applicable to an inclined flat plate having an inclination in the range of $15° \leqslant \gamma \leqslant 90°$ if the definition of the Grashof number is modified by replacing the gravity g with $g \sin \gamma$, the effective gravity component along the surface of the flat plate.

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